Teacher Lesson Pack
Lines and Angles

Suitable for Gr. 6-9

Sir Cumference and the Great Knight of Angleland

By: Cindy Neuschwander, Charlsebridge Publishing, ISBN: 1570911525

Read the book to the students. Ask them to keep a written record of different ideas about lines and angles throughout the story. After reading the story, discuss the ideas that they noted and have the students write the definitions and ideas in their own words in their dictionaries. The class could also generate a single set of definitions for a large poster in the classroom. Some of the terms that should be included: right angle, obtuse angle, acute angle, straight angle, and degrees.

Next, ask the students to draw an angle that is not a right, obtuse, acute or straight angle. Use their drawings to initiate a discussion about reflex angles, leading to their writing of a definition in their dictionaries. Also be looking for the suggestion of a 0° angle. 0 is often a difficult concept for students in other areas of mathematics, for example in fractions, so the more exposure they have to the idea and use of 0 the better. Reflex angles can also be explored using the fraction blocks (the chevron piece).

Finally, have the students play the Splat Degree game found at http://www.sunburst.com/resources/game/index.cfm. (This site is also linked to the Evergreen Curriculum – check for the Pink WWW Resources Button) If you do not have the computers or internet hook-up necessary to play this game, challenge the students to come up with a version of it that does not require a computer. Even if they can access it on the computer, this is a good challenge to give the students.

Some of the vocabulary found in the book:

- diameter
- circumference
- radius
- circle/circular
- obtuse
- right angle
- straight angle
- degrees
- acute angle
- protractor
- parallel lines
- 360

Objectives:

G/M-1 recognize, draw, name, and describe or define a) parallel and perpendicular lines c) right angles, acute angles, obtuse angles d) straight angles, adjacent angles, reflex angles e) complementary angles, supplementary angles, congruent angles, *vertically opposite angles*

G/M-2 identify and compare sizes of angles in the environment as seen in a variety of orientations and line segment lengths

G/M-5 draw an estimate of an angle given in degrees

G/M-6 measure an angle using a a) circular protractor b) semi-circular protractor with dual scale

G/M-7 use a protractor and a ruler to a) draw an angle of a given size b) copy a given angle

G/M-9 recognize that the sum of the interior angles of a triangle is equal to 180°, and the sum of the interior angles of a quadrilateral is 360°. (introduction only)

Angles in Pattern and Fraction Blocks

Materials:

- Paper
- Pattern and fraction blocks
- Protractor

Objectives:

G/M-1 recognize, draw, name, and describe or define c) right angles, acute angles, obtuse angles d) straight angles, adjacent angles, reflex angles

G/M-2 identify and compare sizes of angles in the environment as seen in a variety of orientations and line segment lengths

G/M-5 Draw an estimate of an angle given in degrees

G/M-6

Directions:

Using the orange square with 90° angles, determine the measure of all the other angles found in the different pattern blocks. Write about how you determined each one, and sketch it beside your explanation. You will be asked to share some or all of your approaches with the class after.

Adaptations/Expectations:

- Grade 6: Have the students trace and/or sketch each different angle in the pattern blocks set. They can then order them in increasing size. The students can also be asked to measure the angles using their protractor.
- Grade 8 and 9 can be asked to determine the measure of the angles, using the 90° of the orange square shape, using their knowledge of quadrilaterals ad triangles, and by comparing the pattern block and fraction block shapes.

String Art

Materials:

- Geoboards
- A variety of colours of string (or elastic bands)
- MIRA

Objectives:

G/M-1 recognize, draw, name, and describe or define a) parallel and perpendicular lines b) diagonals of a polygon c) right angles, acute angles, obtuse angles d) straight angles, adjacent angles, reflex angles e) complementary angles, supplementary angles, congruent angles, vertically opposite angles f) alternate interior angles, alternate exterior angles, transversal, corresponding angles, same side interior angles

G/M-2 identify and compare sizes of angles in the environment as seen in a variety of orientations and line segments

G/M-6 measure an angle using a a) circular protractor b) semi-circular protractor with dual scale

G/M-7 use a protractor and a ruler to a) draw an angle of a given size b) copy a given angle

Description of task:

Have the students use white string to create a string art pattern on their geoboard. This can also be done on a traditional string art board set-up, but it is not necessary. After the students have completed their design, give them a set of instructions that are designed to have the students focus on particular lines and angles that you have chosen. For example, you might say, using

- Green to show a pair of corresponding angles
- Red to show a reflex angle
- Blue to show parallel lines
- Yellow to show diagonals in a polygon
- And so on…

The students could then create a legend for their design and the designs could be displayed in the school.

Adaptations:

- Have the students identify parallel and perpendicular lines using a MIRA
- Have the students copy some or all of their string art design using a compass and straightedge and/or MIRA.
- Cork board could be used instead of the geoboards. Have the students place a sheet of white paper on the cork board and then use straight pins to both secure the paper and to wrap string around to create their design. Students can then use markers on the string to highlight the different geometrical terms that they are asked to find. This would transfer to the white sheet of paper which could later be removed for discussion/assessment.

Complementary and Supplementary Angles – Introduction

Materials:

- Brads
- Straight angle and right angle templates
- Paper
- Protractor

Objectives:

G/M-1 recognize, draw, name, and describe or define e) complementary angles, supplementary angles, congruent angles, *vertically opposite angles*
G/M-5 draw an estimate of an angle given in degrees
G/M-7 use a protractor and a ruler to a) draw an angle of a given size
G/M-8 determine the measures of vertically opposite, supplementary, and complementary angles when one angle is given

Description:

The purpose of this activity is to have the students physically manipulate angles to develop a better understanding of complementary and supplementary angles.

Using a template that is made out of card, the students attach a third line to the vertex of the angle using a brad, and then move the third line back and forth, measuring the resulting angles and determining a relationship.

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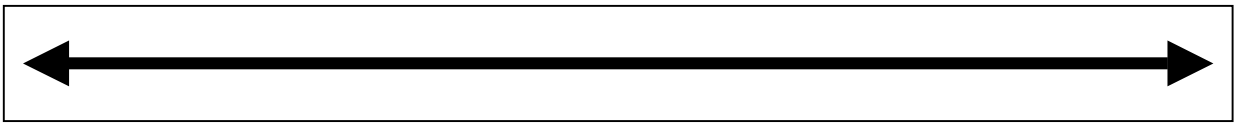
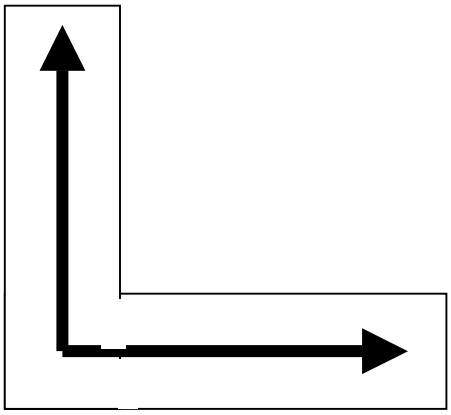




Complementary and Supplementary Angles – Card Game

Materials:

- Cards numbered 0° to 180° plus any extras desired

Objectives:

G/M-1 recognize, draw, name, and describe or define e) complementary angles, supplementary angles, congruent angles, *vertically opposite angles*

G/M-5 draw an estimate of an angle given in degrees

G/M-7 use a protractor and a ruler to a) draw an angle of a given size

G/M-8 determine the measures of vertically opposite, supplementary, and complementary angles when one angle is given

Description:

Provide each group with a set of cards that are to be shuffled. The game can be played in a variety of ways.

1. Each player dealt 6 cards. Taking turns, each player asks someone else if they have a particular card. The card that is requested must be one that is either a supplement or a complement to a card in the player's hand. If the other player asked has the card, they must give it to the requester. If not, the player requesting draws one card. If the card is a complement or a supplement to a card in the player's hand they may lay the two cards down. If not, the player keeps the card in their hand and play moves to the next player. The game is completed when one player has laid down all of their cards as supplement or complement pairs.
2. Each player dealt 6 cards. The goal of this version is also to collect pairs of angle cards that are complements or supplements, but in this version they do not request cards from other players. Rather they draw a single card at the start of each turn, or take the last previously discarded card. The player ends their turn by discarding one card. When a player has all of the angle cards in his/her hand matched with as complements or supplements the game is completed.
3. Spread the cards out over the table face down. Players take turns flipping over two cards. If the cards are complementary or supplementary angles, the player removes the two cards and draws again. If they are not complementary or supplementary, the cards are flipped back over and the next player starts their turn. The game ends after a set period of time, or once there are no remaining pairs. The player who has collected the most pairs is the winner.

Assessment:

- In any of the versions of the game, students with remaining angle cards could be asked to either construct the angle and its complement and/or supplement, or to sketch them (depending on the grade level)
- Have the students each draw one card and then state and sketch the supplement and/or complement of the angle.

0°	5°	10°
15°	20°	25°
30°	35°	40°
45°	50°	55°

60°	65°	70°
75°	80°	85°
90°	95°	100°
105°	110°	115°

120°	125°	130°
135°	140°	145°
150°	155°	160°
165°	170°	175°

180°	FREE	FREE
15°	20°	25°
30°	35°	40°
45°	50°	55°

60°	65°	70°
75°	80°	85°
90°	95°	100°
105°	110°	115°

120°	125°	130°
135°	140°	145°
150°	155°	160°
165°	170°	175°

180°	FREE	FREE
0°	5°	10°
0°	5°	10°
15°	20°	25°

180°	FREE	FREE
30°	35°	40°
45°	50°	55°
DRAW 2	DRAW 2	DRAW 2

60°	65°	70°
75°	80°	85°
90°	95°	100°
105°	110°	115°

120°	125°	130°
135°	140°	145°
150°	155°	160°
165°	170°	175°

Adjacent Angles – What a Concept

Using the templates given, or pictures you have made or found, carry out a concept attainment with the students about adjacent angles. To do this, have a location for examples to be posted and another for non-examples. Select one picture, show it to the students and say "this is a (yes/no)" and place it in the appropriate area. Select another and repeat the process. This should be done at least 3 or 4 times without the students guessing or speaking out. Be sure to include both examples and non-examples.

On a large record sheet or overhead, ask the students to give characteristics that they think a yes will have (or that it will not have). Continue showing more examples and non-examples, and have the students vote using thumbs up, thumbs down, or not sure (thumb sideways). For the first few, it is best to just allow voting – not discussion. Decide the vote, but place the picture in the appropriate location. Have the students edit the characteristics list as you go along – removing and adding as they see more examples and non-examples.

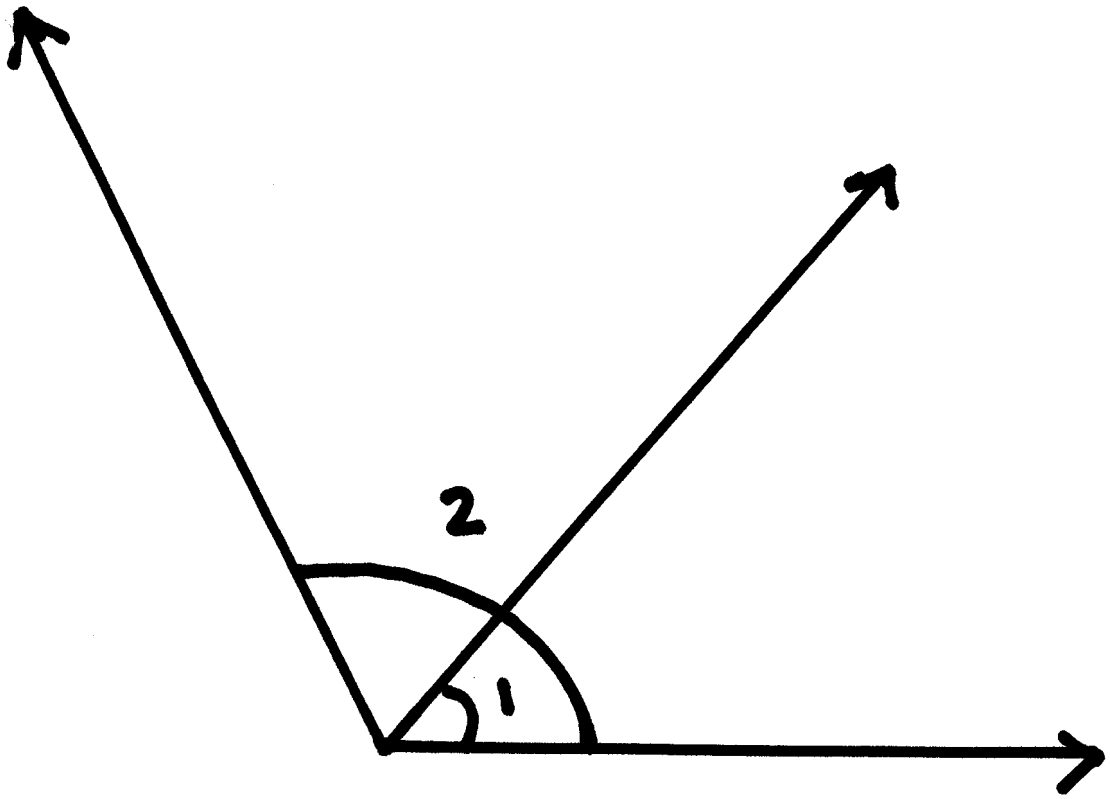
When the students have captured the characteristics of adjacent angles, and do not have extraneous ones, they are ready to be given the term "adjacent angles" and to write and illustrate it in their dictionaries.

Objectives

G/M-1 recognize, draw, name, and describe or define d) straight angles, adjacent angles, reflex angles

Resource:

This is a Yes: Concept Attainment, SIDRU



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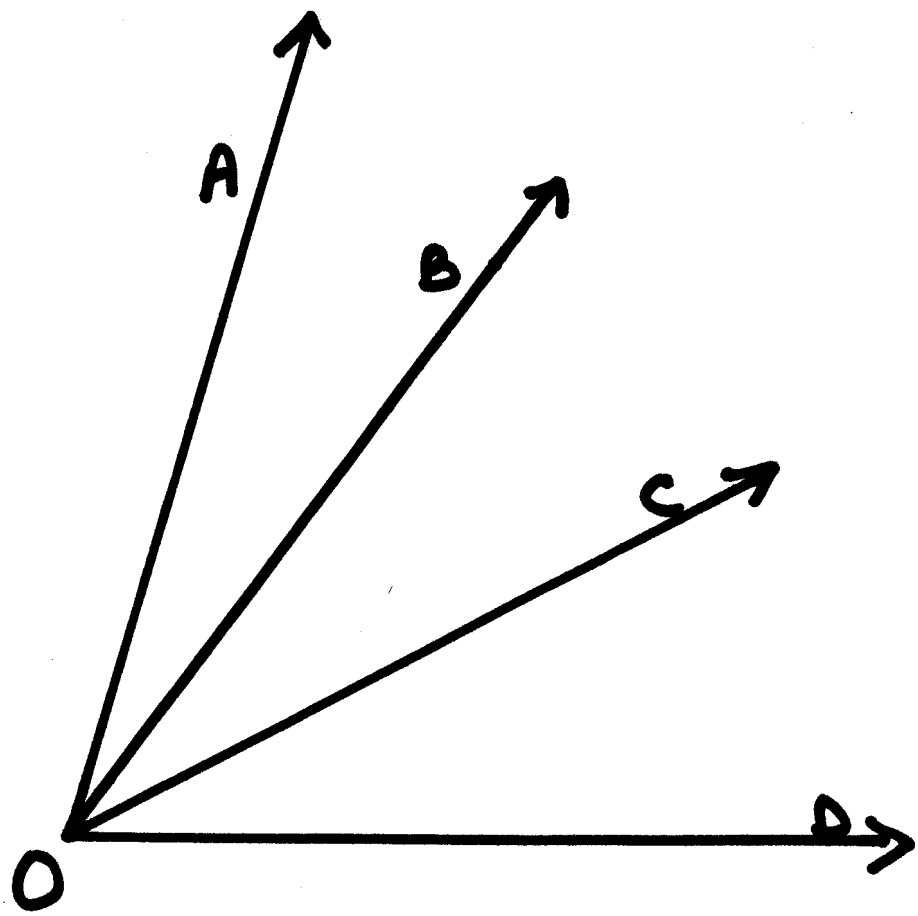
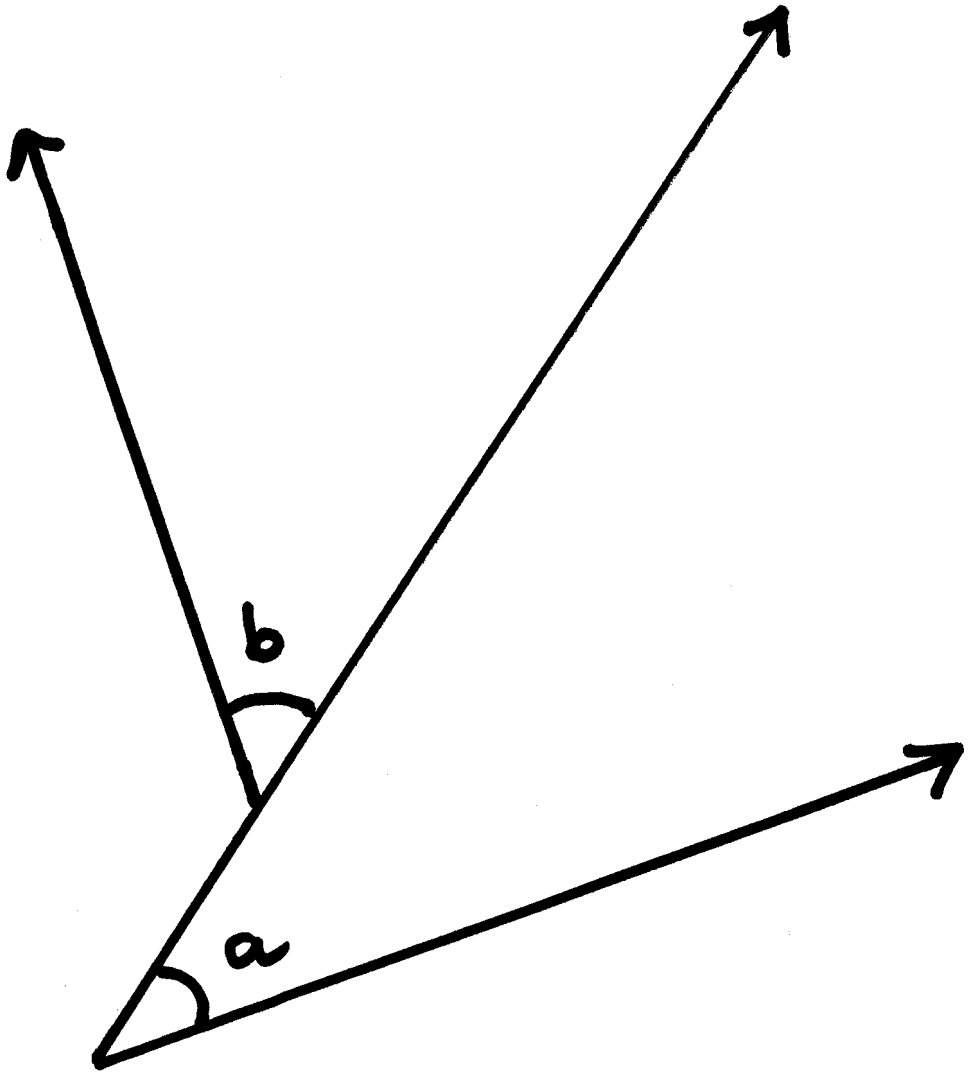
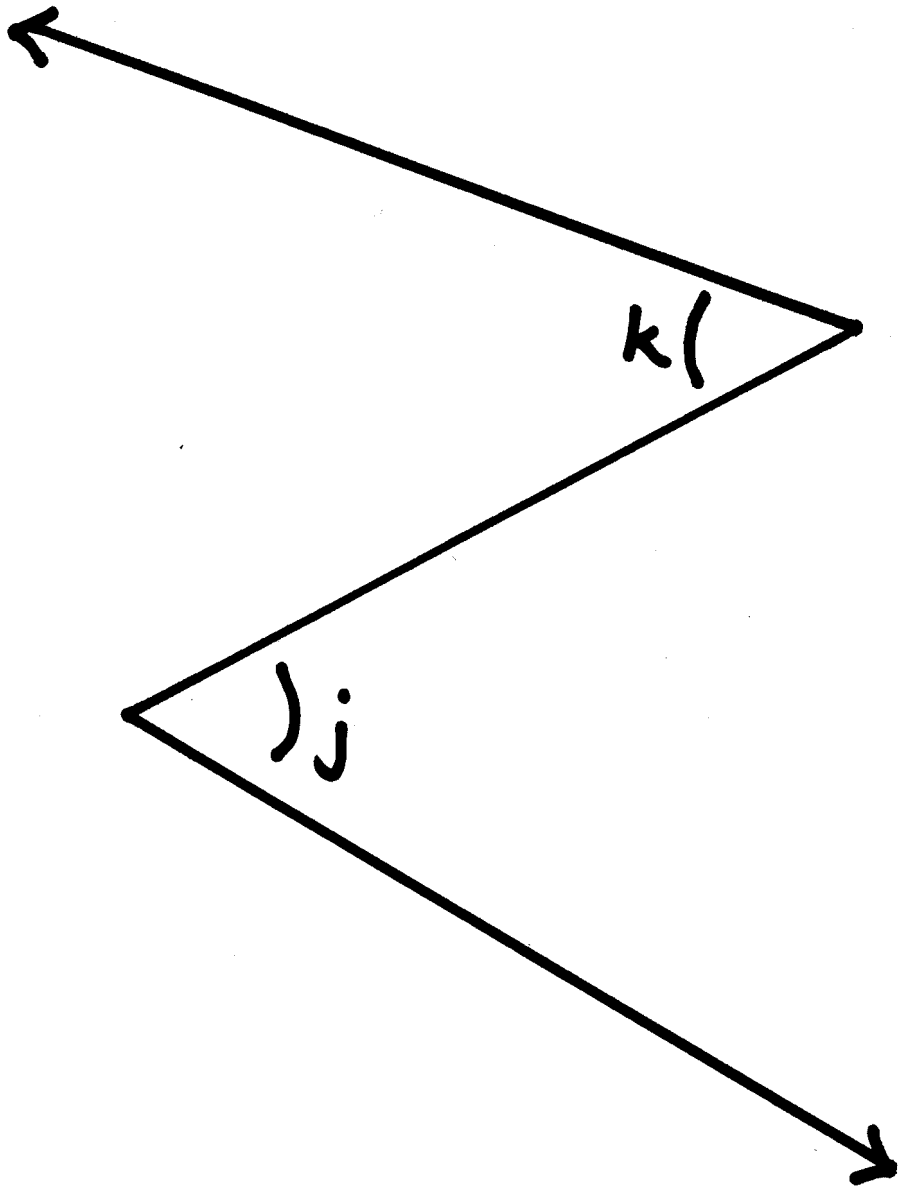


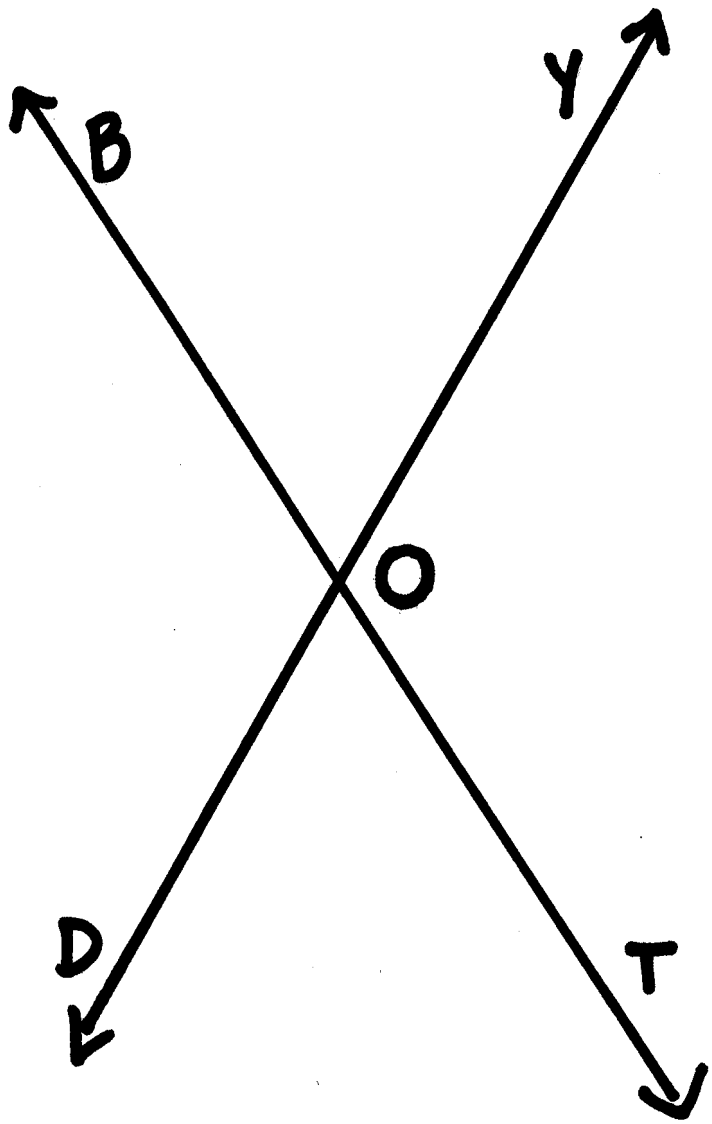
$\angle AOC$ and $\angle BOD$



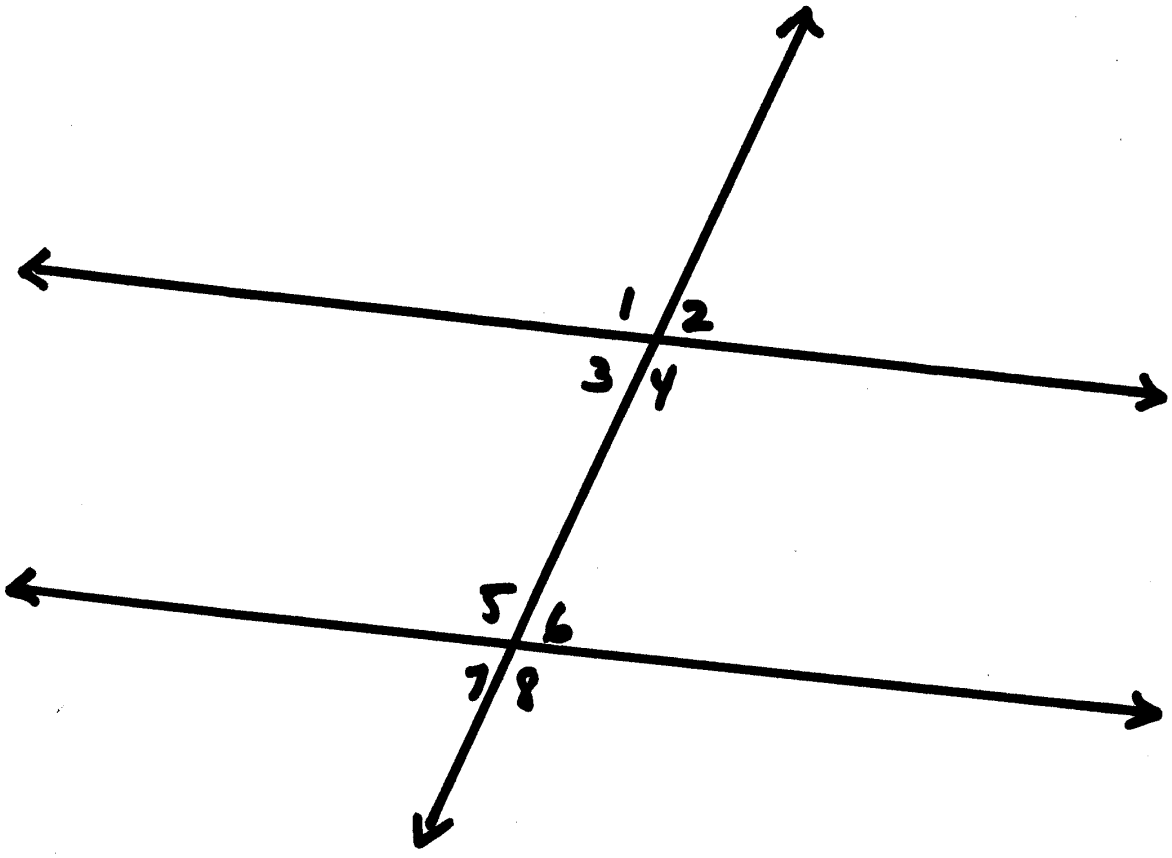
$\angle a$ and $\angle b$



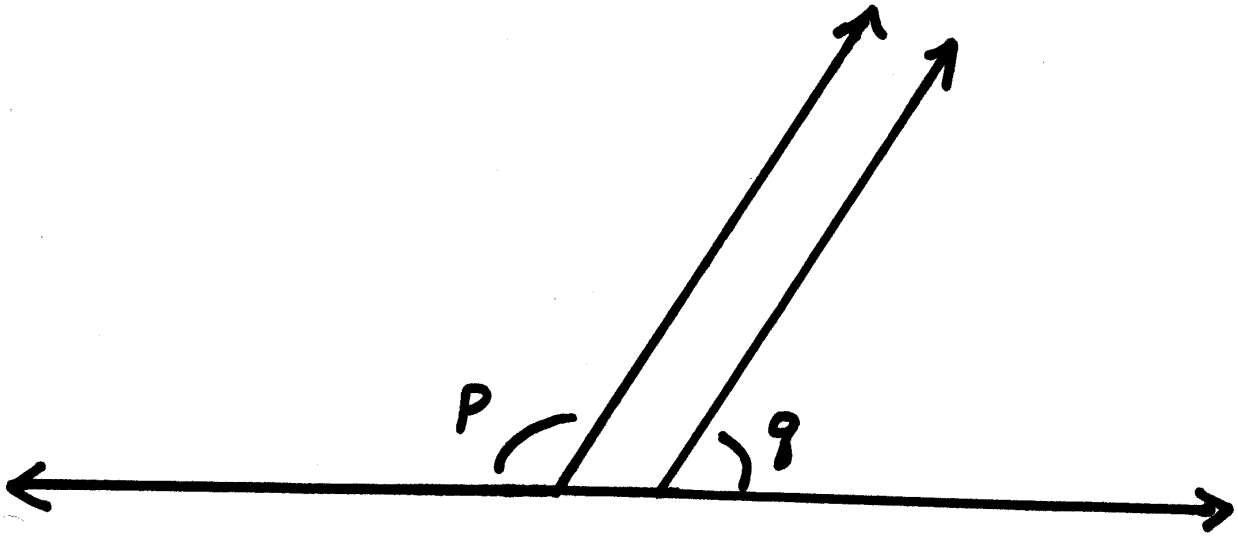
$\angle k$ and $\angle j$



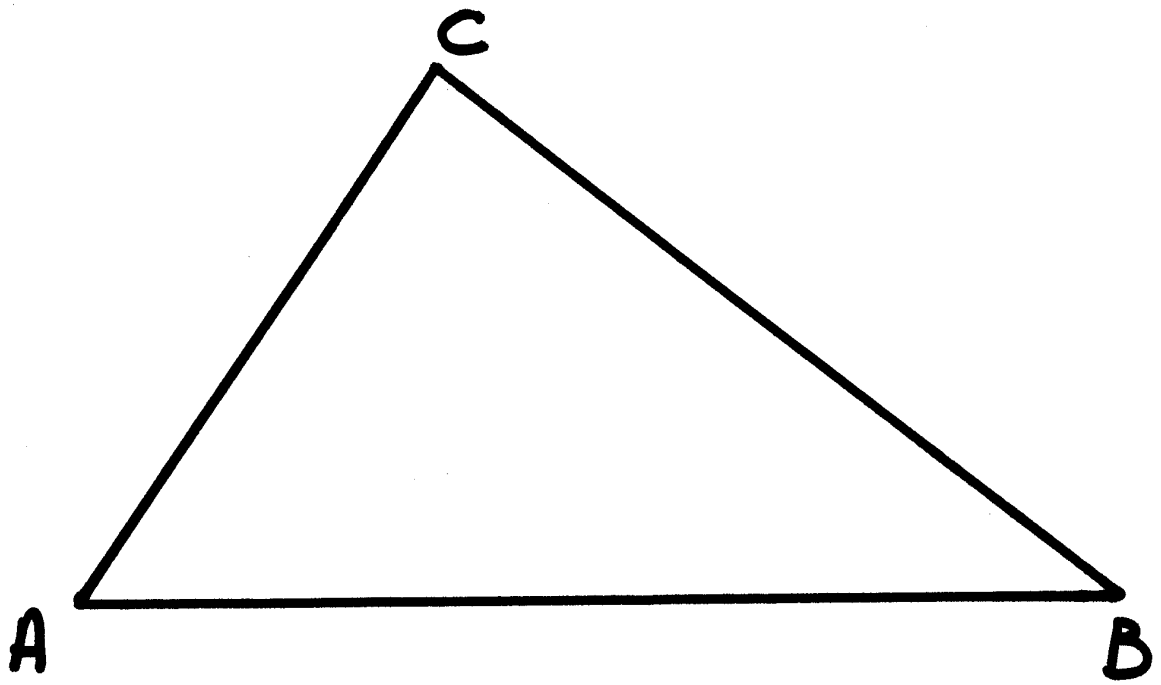
$\angle BOY$ and $\angle DOT$



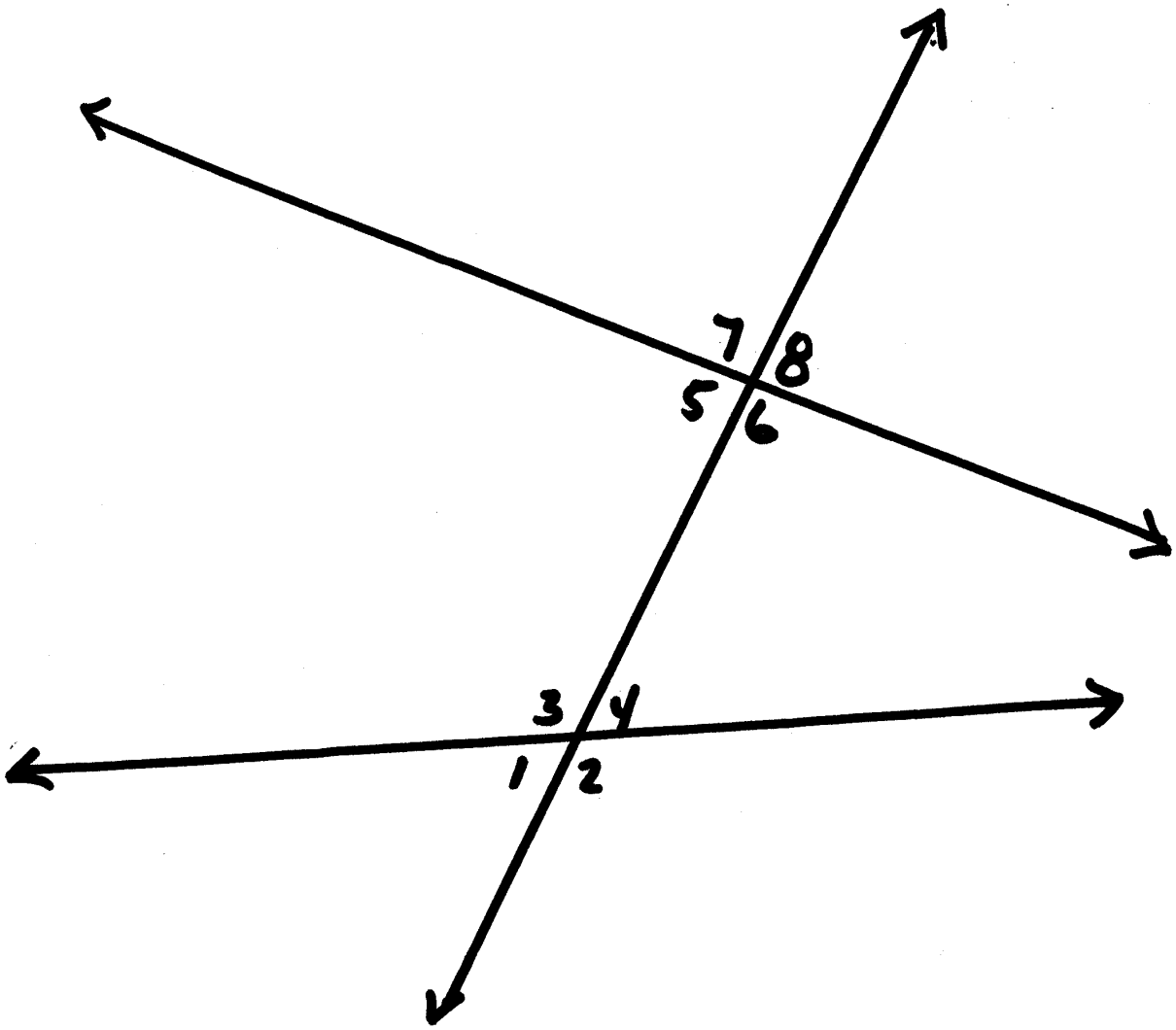
$\angle 3$ and $\angle 5$



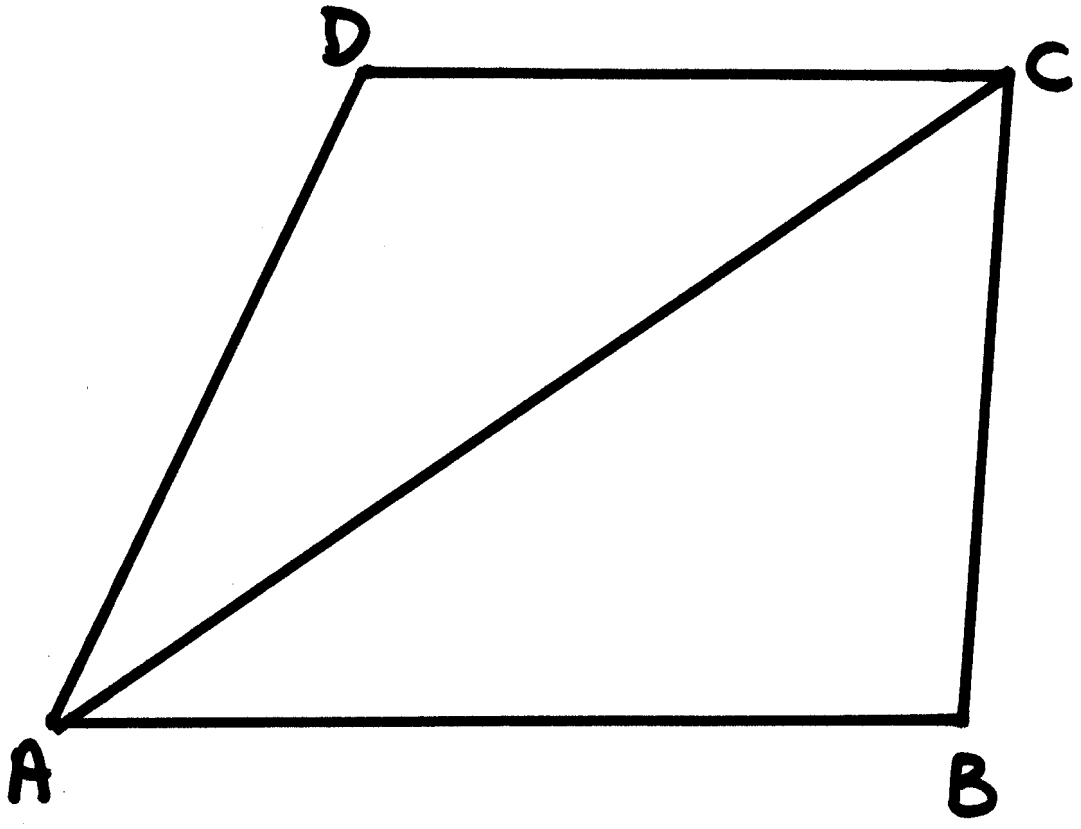
$\angle p$ and $\angle q$



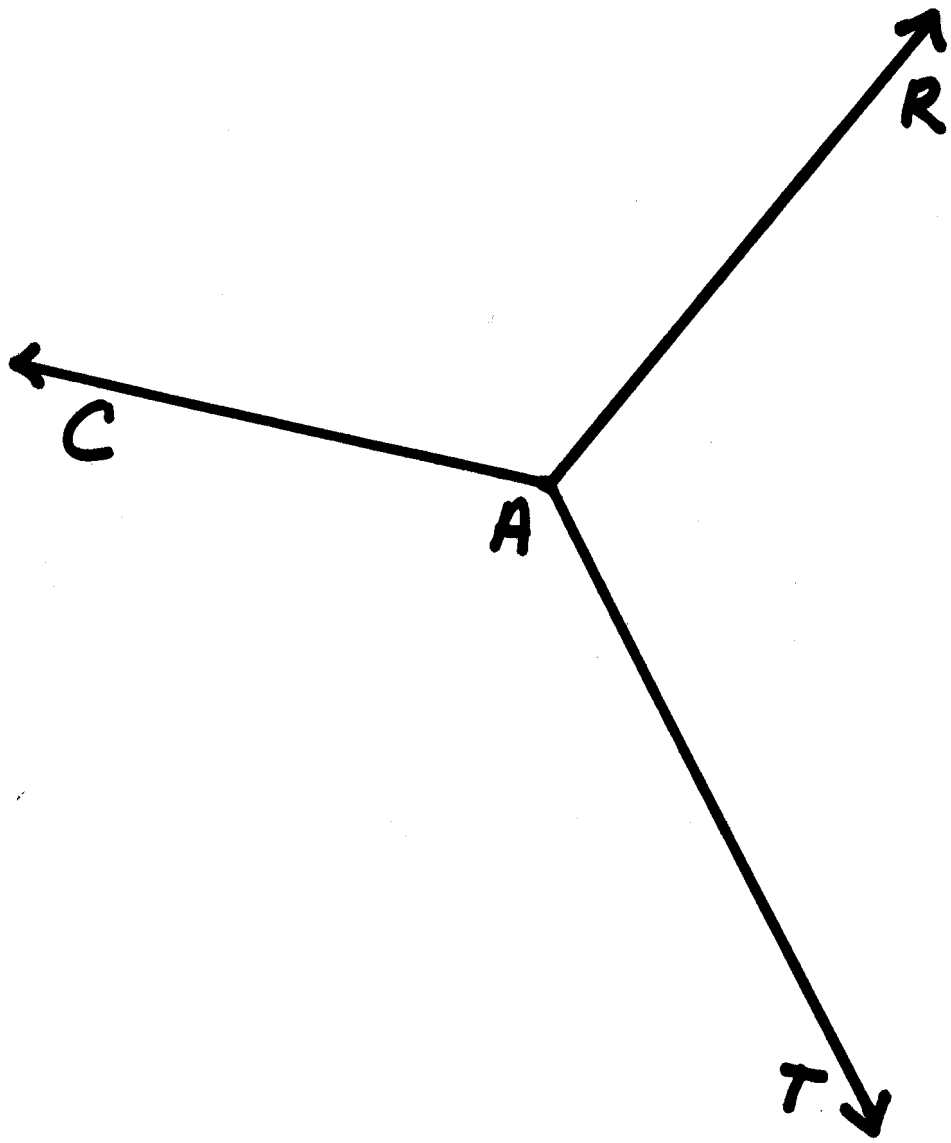
$\angle CAB$ and $\angle ABC$



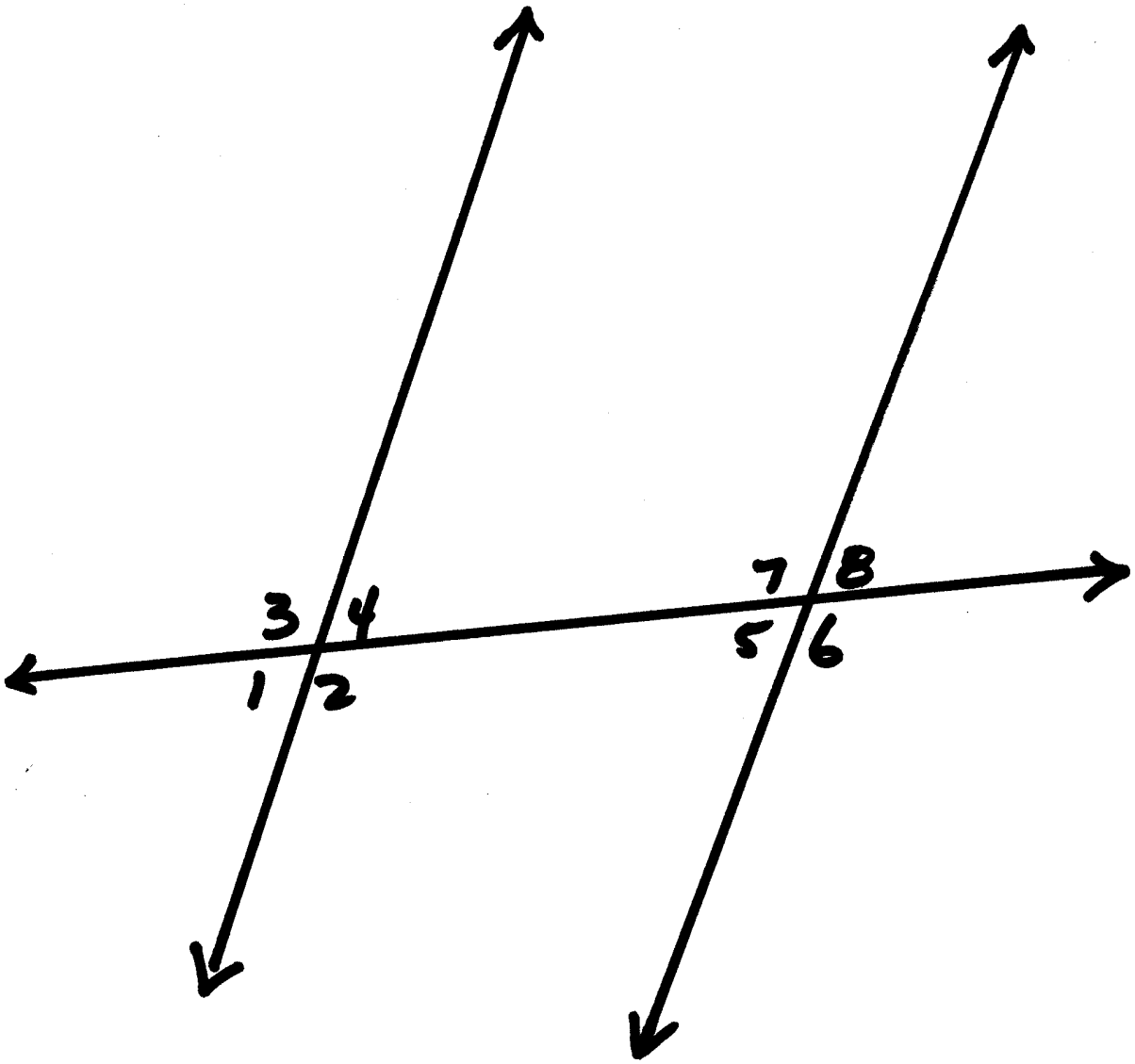
$\angle 5$ and $\angle 7$



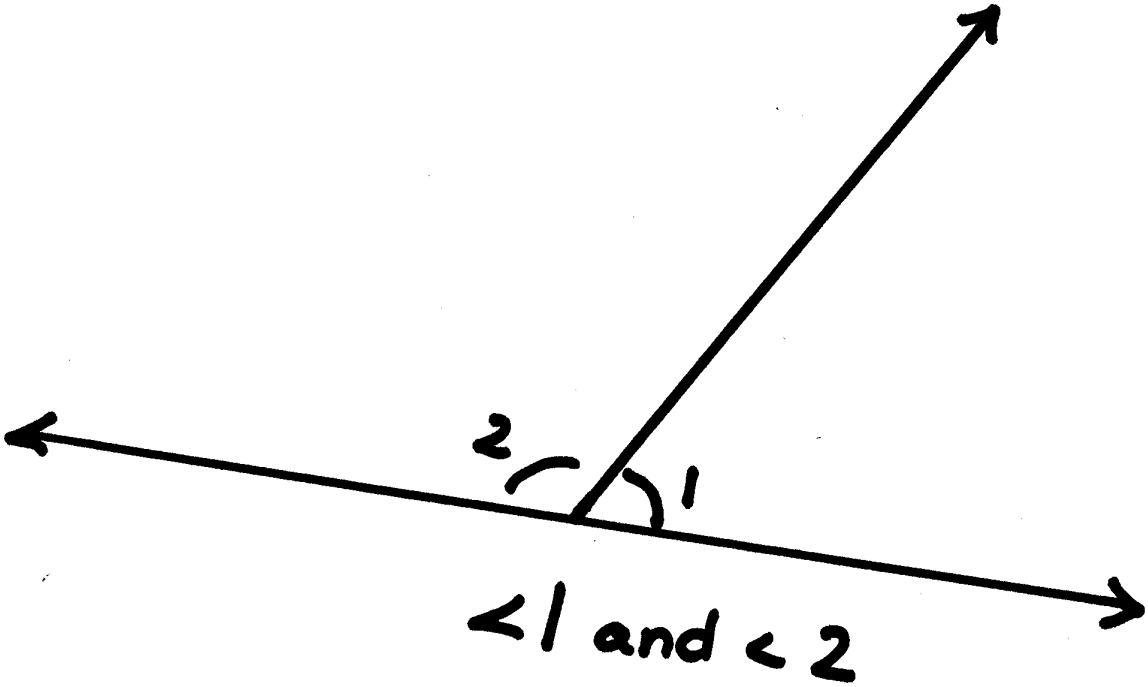
$\angle DAC$ and $\angle BAC$



$\angle CAR$ and $\angle TAR$



$\angle 2$ and $\angle 4$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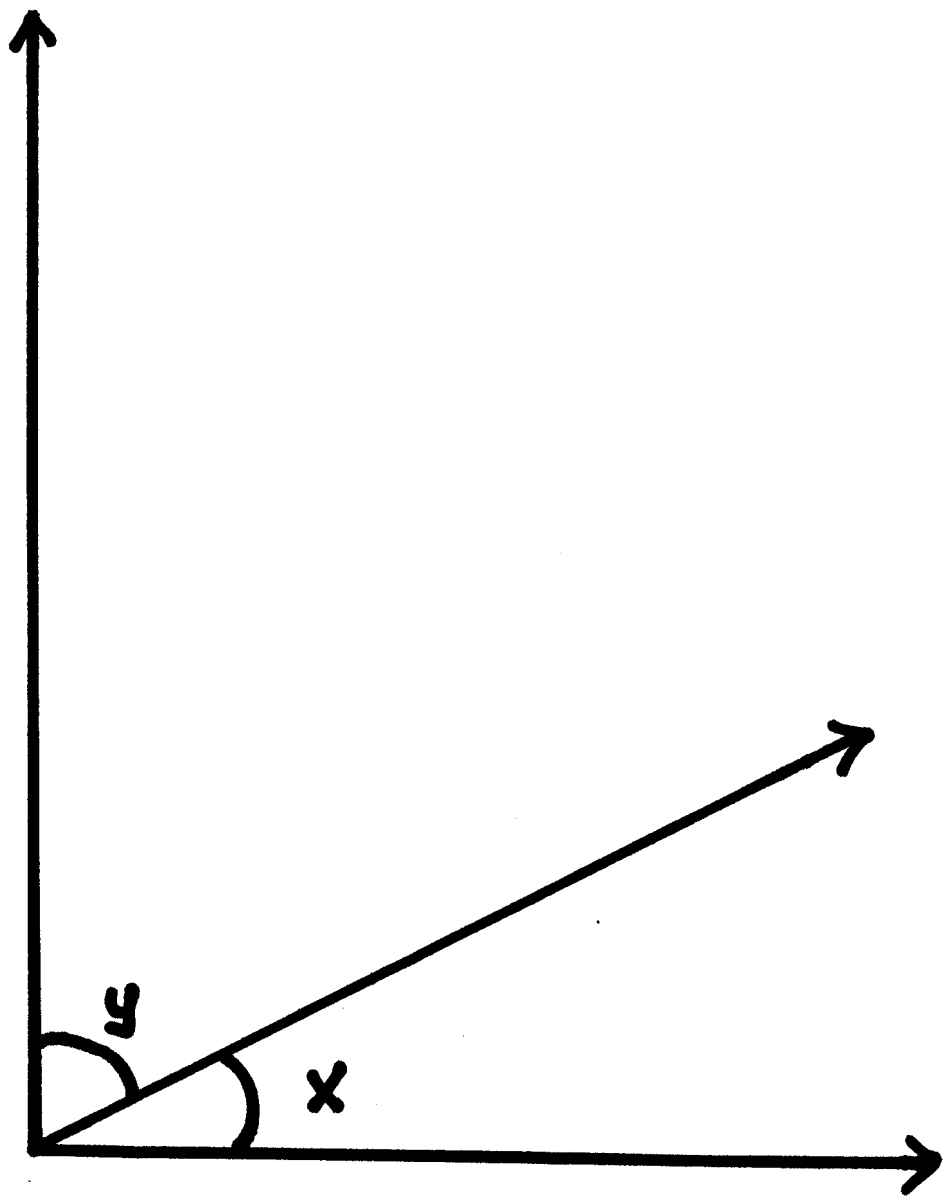


$\angle x$ and $\angle y$

Parallel Lines. Perpendicular Lines and Transversals

Materials:

- Maps, pictures
- Markers
- Paper
- Scissors

Objectives:

G/M-1 recognize, draw, name, and describe or define a) parallel lines, perpendicular lines f) alternate interior angles, alternate exterior angles, transversal, corresponding angles, same side interior angles

Description:

1. Have the students use the term "transverse" in context. For example, I "transversed" the street. Discuss the definition of the term. Introduce the term transversal and relate it to transverse.
2. Draw a pair of intersecting lines and identify the transversal.
3. Draw a pair of non-parallel lines and a transversal. Have the students identify all possible transversals.
4. Using a map or pictures, have the students highlight with different coloured markers lines which are parallel, perpendicular, or neither. Also ask the students to highlight lines which are transversals.
5. Have the students create paper dolls/paper chains and describe/highlight different lines that they can see in their resulting creations – parallel lines, perpendicular lines, transversals. The paper chain/doll designs are made by repetitively folding paper in half, then cutting along the edge(s) that have the open side(s) on them. Legal paper works well.

Angles formed by Transversals

Materials:

- Diagram Handout
- Angle pairs and counter example cards
- Enlarged alternate diagram
- MIRA
- Compass, straightedge

Objectives:
G/M-1 recognize, draw, name, and describe or define e) *complementary angles,* supplementary angles, congruent angles, vertically opposite angles g) alternate interior angles, alternate exterior angles, transversal, corresponding angles, same side interior angles
G/M-3 solve problems involving angles including those formed by parallel lines cut by a transversal
G/M-4 construct a line parallel to a given line through a point not on the line using paper folding, mira, and/or compass and straightedge

Description of Task:
Have the students work in small groups or pairs. Provide each group with a copy of the diagram with parallel lines and two non-parallel transversals and numbered angles as well as a card with one angle set on it and a counter example on the back of the card.

The students' first task is to identify the characteristics defining the pairs of angles that are on their card. Next, the students are to consider the pair of angles that is given as a counter-example on the back of the card and to see what characteristic(s) this particular pair of angles does not have. If they cannot see the contradiction, they must go back and reconsider their defined characteristics so that there is a contradiction.

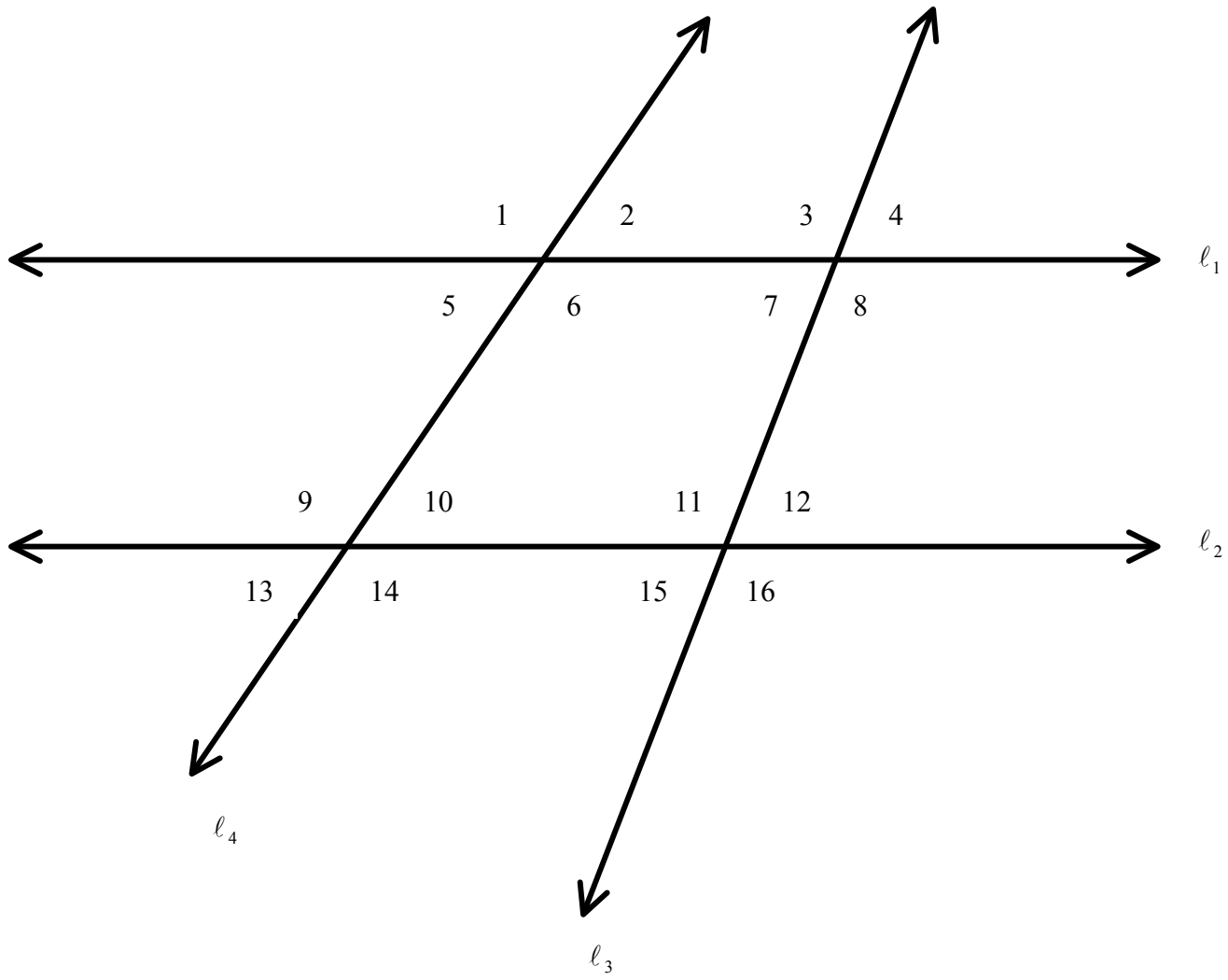
Once all of the groups feel that they understand the type of angle that they have been given examples above, put up the second diagram that is labeled using letters and that has been drawn with a different orientation. First, have the class discuss what lines, if any are parallel, and which they would call transversals and why. Next, ask the students to identify all pairs of angles in this new diagram (using the letters) that are the same type as what they have been looking at.

Following the completion of each group creating their list, select a pair of angles and ask the students which groups had that type of angle. Have those groups explain how they know (give the characteristics). It may be beneficial to also post the original diagram for the students to refer to. As a class, write out the characteristics of the pair of angles and give a few minutes for the groups who had worked with other types of pairs to identify all pairs of angles on the new diagram that have the same characteristics. Finally, give the students the name for the type of angles (vertically opposite, adjacent, etc.). Allow the students some time to discuss how they would remember the name and/or recognize the angles in other situations. Also ask the students if they could identify angles that are of the same type, but where there aren't parallel lines.

Repeat the process with the other types of angles.

Adaptations:

- Grade 8 and 9 students could be asked to focus on vertically opposite angles. Can they draw them in different contexts? How are they related to each other? The students could be asked to measure, cut out, paper fold, and use the MIRA to explore this relationship.
- Grade 9 students could be asked to create a set of parallel lines using paper folding, the MIRA, or compass and straightedge and then to identify the different pairs of angles. Using the compass, paper folding, and/or the MIRA the students could then be asked to look for relationships. These relationships would then be discussed within small groups, and each small group could be assigned one type of angle to report about their findings.
- Grade 9 students could be asked to construct non-parallel lines with a transversal. Have them write a journal entry about what they are able to determine about the different types of angles in this situation. Have a group discussion about the consequences of lines not being parallel on the angle pairs, as well as the reverse condition.
- Both grade 8 and 9 students can be asked to identify examples of each type of angle from within their environment. This could include physical objects, pictures in the media, photos they have taken, or drawings they have created. Students could use markers to highlight pairs of angles and to write a description about the type of angles, the relationship between the different lines, etc.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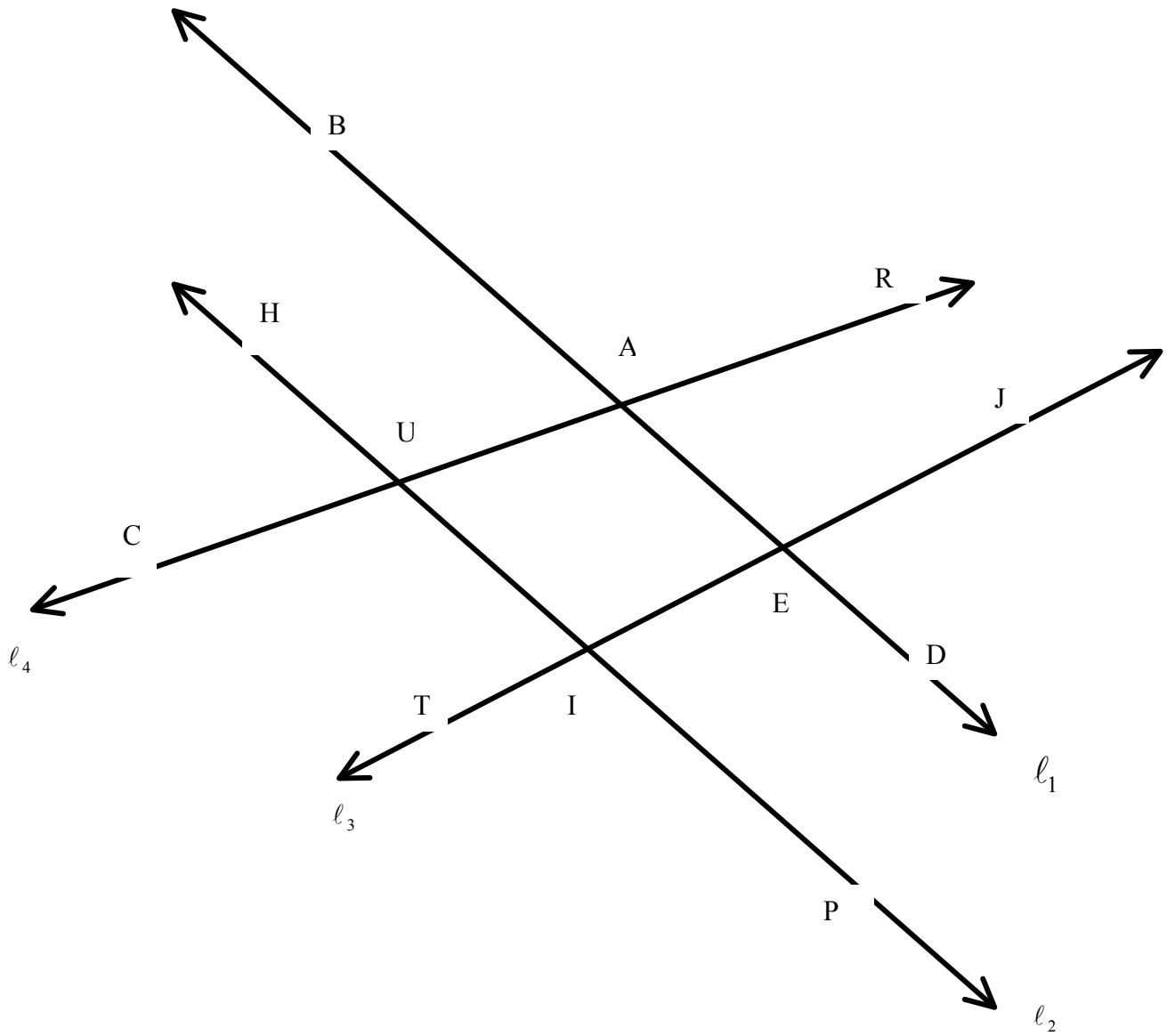
Angle Pair Set #1 <1 and <6 <2 and <5 <3 and <8 <4 and <7 <9 and <14 <10 and <13 <11 and <16 <12 and <15	Angle Pair Set #2 <1 and <2 <9 and <10 <1 and <5 <9 and <13 <2 and <6 <10 and <14 <5 and <6 <13 and <14 <3 and <4 <11 and <12 <3 and <7 <11 and <15 <4 and <8 <12 and <16 <7 and <8 <15 and <16
Angle Pair Set #3 <5 and <10 <6 and <9 <7 and <12 <8 and < 11	Angle Pair Set #4 <1 and <9 <2 and <10 <3 and <11 <4 and <12 <5 and <13 <6 and <14 <7 and <15 <8 and <16
Angle Pair Set #5 <1 and <14 <2 and <13 <3 and <16 <4 and <15	Angle Pair #6 < 5 and <9 <6 and <10 <7 and <11 <8 and <12

Alternate Example #1 <1 and <14	Alternate Example #2 <3 and <8
Alternate Example #3 < 6 and <11	Alternate Example #4 <1 and <10
Alternate Example #5 <2 and <14	Alternate Example #6 <5 and <11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Parallel Lines and Angles Challenge

Materials:

- Worksheet
- Overhead of worksheet
- Straightedge and compass

Objectives:

The objectives covered will depend on the approach that the students take to solving the problems – especially in the case of the second problem.

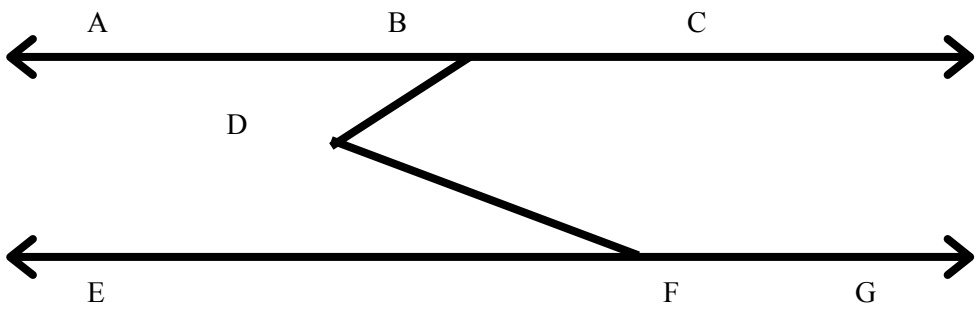
Description:

Working alone or in pairs, have the students explore all of the angle relationships that they can find and then specifically look for the relationship between the three angles stated for the problem. The students are allowed to make constructions on the diagrams, but since they are only sketches, measurement of the angles will not lead to the desired conclusions.

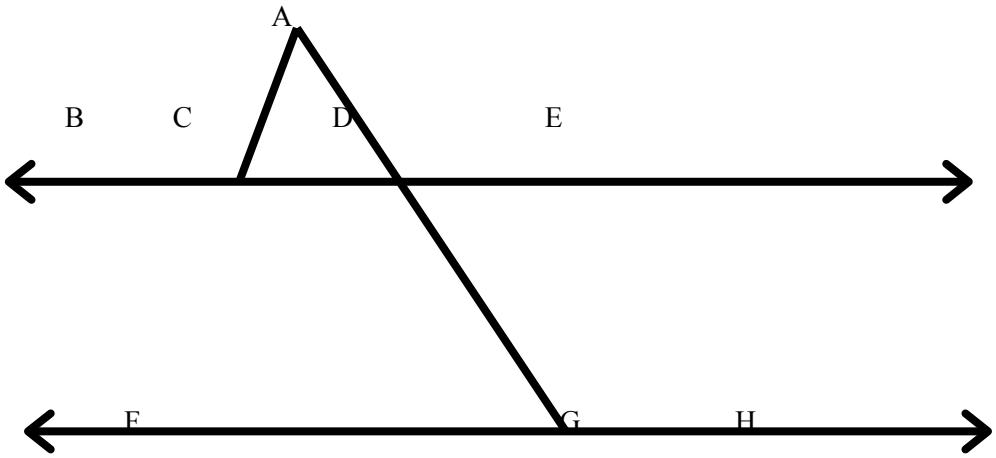
If the students are struggling with the problems, a hint might be provided in the first such as " where else might you want to draw a parallel line and what would it be parallel to?"

Be sure to provide an opportunity for the students to share their results with the class, both to confirm their understanding, but also to highlight any differences in approach that may have been used.

Find a relationship between the angles indicated for each diagram below. Explain how you know this relationship.



<ABD, <BDF, and <DFE $\left(\overleftrightarrow{AC} \text{ is parallel to } \overleftrightarrow{EG}\right)$



<DGH, <ACD, and <CAD $\left(\overleftrightarrow{BE} \text{ is parallel to } \overleftrightarrow{FH}\right)$

Tessellations to find the Sum of Angles

Materials:

- Overhead pattern blocks
- Paper
- Water soluable markers
- Pattern blocks

Objectives:

G/M-9 recognize that the sum of the interior angles of a triangle is equal to 180°, and the sum of the interior angles of a quadrilateral is 360°.
G/M-10 calculate the value of an unknown angle in a triangle and in a quadrilateral given the measures of the other angles.
G/M-28 identify and draw, using simple figures, a) reflections (flips), rotations (turns), translations (slides) b) a combination of translations, reflections, and rotations
G/M-33 continue or create patterns using translations, reflections, and/or rotations
G/M-35 explain why some shapes tessellate and identify and create shapes that tessellate

Description:

In small groups, have the students each select one of the pattern block shapes that is either a quadrilateral or a triangle. Demonstrating on the overhead, have the participants number the angles in their shape (on up to 10 pieces). The marking of the angle should be done so that the corresponding angles between the pieces have the same mark or number on them. Next, have the participants tessellate their pieces, putting different angles against each other. Next, have the participants trace them, including marking down the angles on the tracing. Then have the participants discuss what they have found regarding the relationship between the angles in the triangle and/or in the different quadrilaterals.

Next, discuss with the participants how this activity could be expanded to different triangles and quadrilaterals that the students would construct to test out their theories about the sum of the angles in both types of figures.

A further extension of this activity would be to, in the case of regular polygons, have the students determine the measure of the angles given what they know about the sum of the angles. Similarly, the students could be asked to construct a triangle or quadrilateral with 2 or 3 known angles respectively, predict what they think the measure of the 3rd or 4th angles should be respectively, and to determine a way to verify this (with or without measuring).

Constructions with Paper Folding

Included is an example of a paper folding (Origami) that makes what is called a Frilled Lizard. The lizard can be unfolded and the fold lines marked. The students can then use the resulting fold lines to identify: angle bisectors, line segment bisectors, parallel lines, transversals, and so on. The students can then be asked to come up with a set of instruction on how to paper fold these lines, line segments, and angles. Students can explore other Origami designs and look at what constructions they can get out of them as well as what they can recognize in the resulting angles and lines.

Constructions with MIRA

Given an opportunity to play and explore, students become very adept at using the MIRA. Simply providing the students with a number of constructions that they should be able to do and asking them to come up with the instructions for how to create them will work. Students can be paired or even in small groups where they do a modified version of a jigsaw – each one responsible for teaching the others how to do a particular construction using the MIRA.

Constructions with Compass and Straight Edge

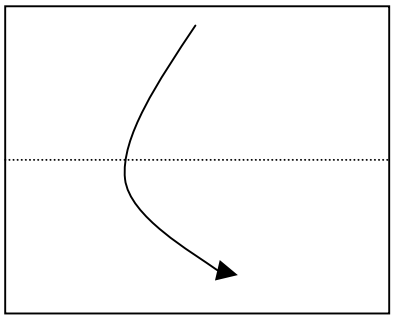
If time permits, just having the students play with a compass and straight edge will often lead to their development of the process for doing various constructions. Sometimes, they require one or two examples to get them started. Often, because only a limited number of constructions are done at any grade the exploration can be given to the students as a future homework assignment.

If time is an issue, the students can also be simply directed on how to do the constructions, with discussions considering why the constructions work. G/M-4 (construct a line parallel to a given line through a point not on the line) can be used to emphasize the relationships that the students have determined about angles resulting from transversals crossing parallel lines (G/M-3). These constructions can be found online and in most middle level textbooks.

Frilled Lizard

Use a 15 cm square paper.

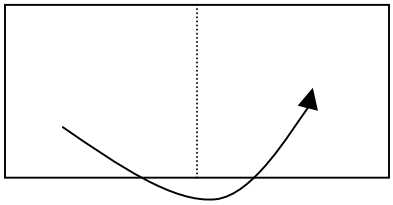
1



Fold in half

2

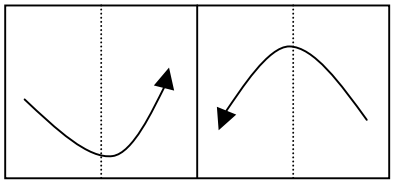
Fold in half



3

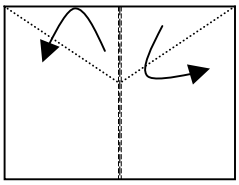
Undo last fold

4



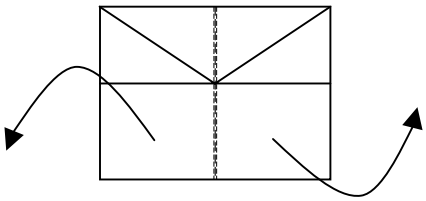
Fold each half in half

5



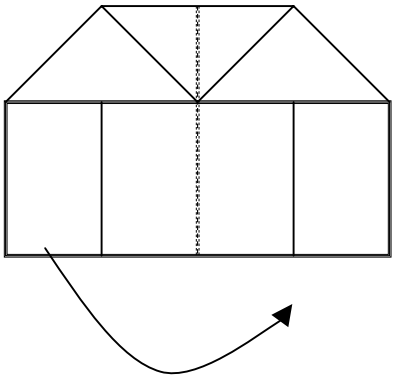
Fold down the corners

6



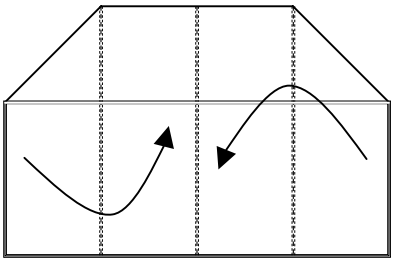
Fold out each outside flap

7



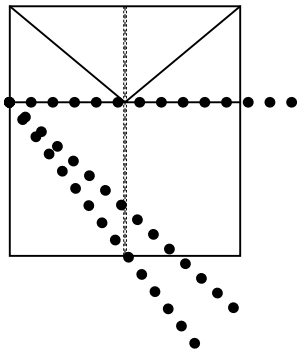
Flip over to other side

8



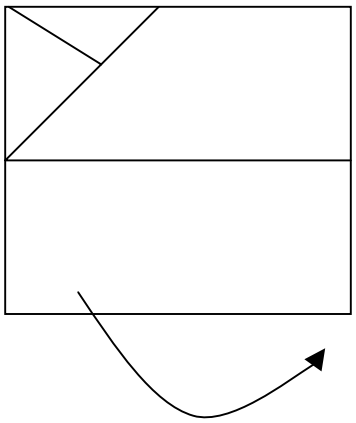
Fold sides in

9



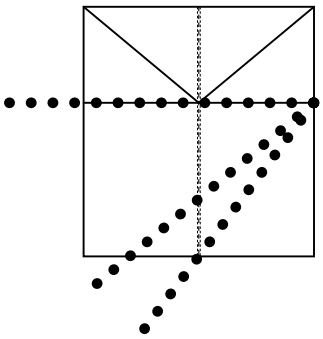
Fold top layer, starting at bottom three times – dotted lines indicate folds – start from bottom and work up

10



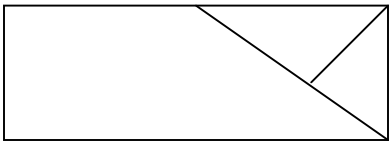
Flip over

11



Repeat step 9 on this side:
Fold top layer, starting at bottom three times – dotted lines indicate folds – start from bottom and work up

12



Put two fingers inside. Push in the center.